流体力学
Fluid Mechanics

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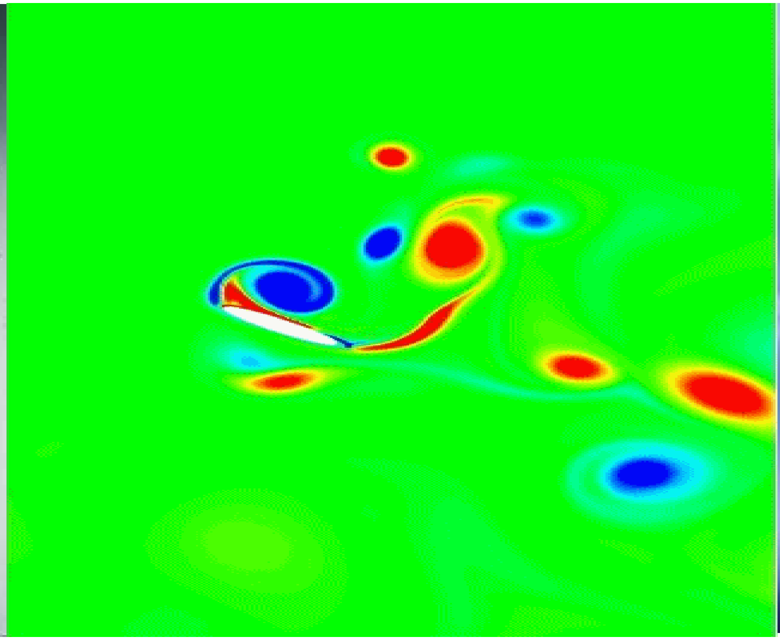
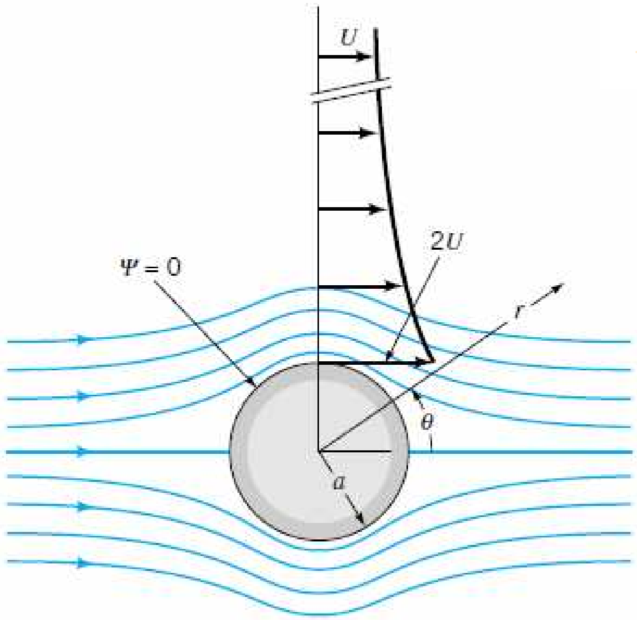
欢迎大家进入奥妙无穷、绚丽多彩的流体力学世界！


参考教材:《流体力学》（第三版）江苏大学 罗惕乾主编 机械工业出版社 2007

《**Fundamentals of Fluid Mechanics**》 **Bruce R. Munson, Donald F. Yong, Theodore H. Okiishi**

Inviscid Flow Around a Circular Cylinder



$$F_x = -\int_0^{2\pi} p_s \cos\theta \, a \, d\theta$$

$$F_y = -\int_0^{2\pi} p_s \sin\theta \, a \, d\theta$$

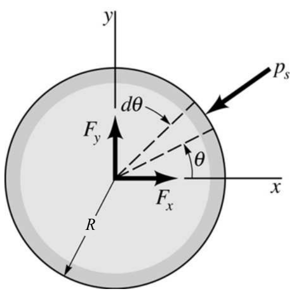
$F_x = 0$ $F_y = 0$

d'Alembert paradox

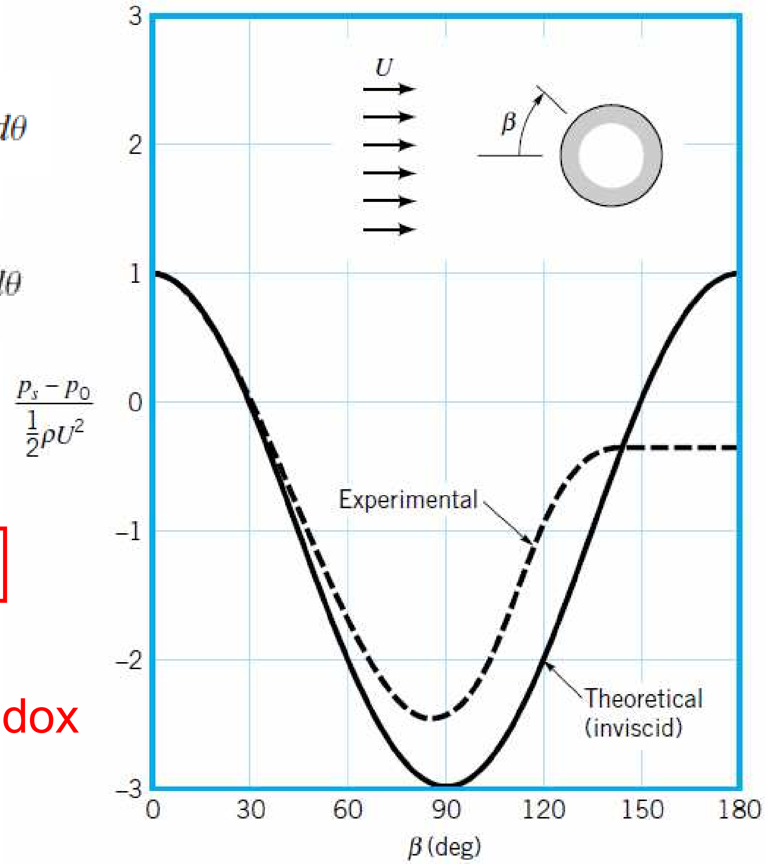
Jean LeRond d'Alembert

(1717-1783)

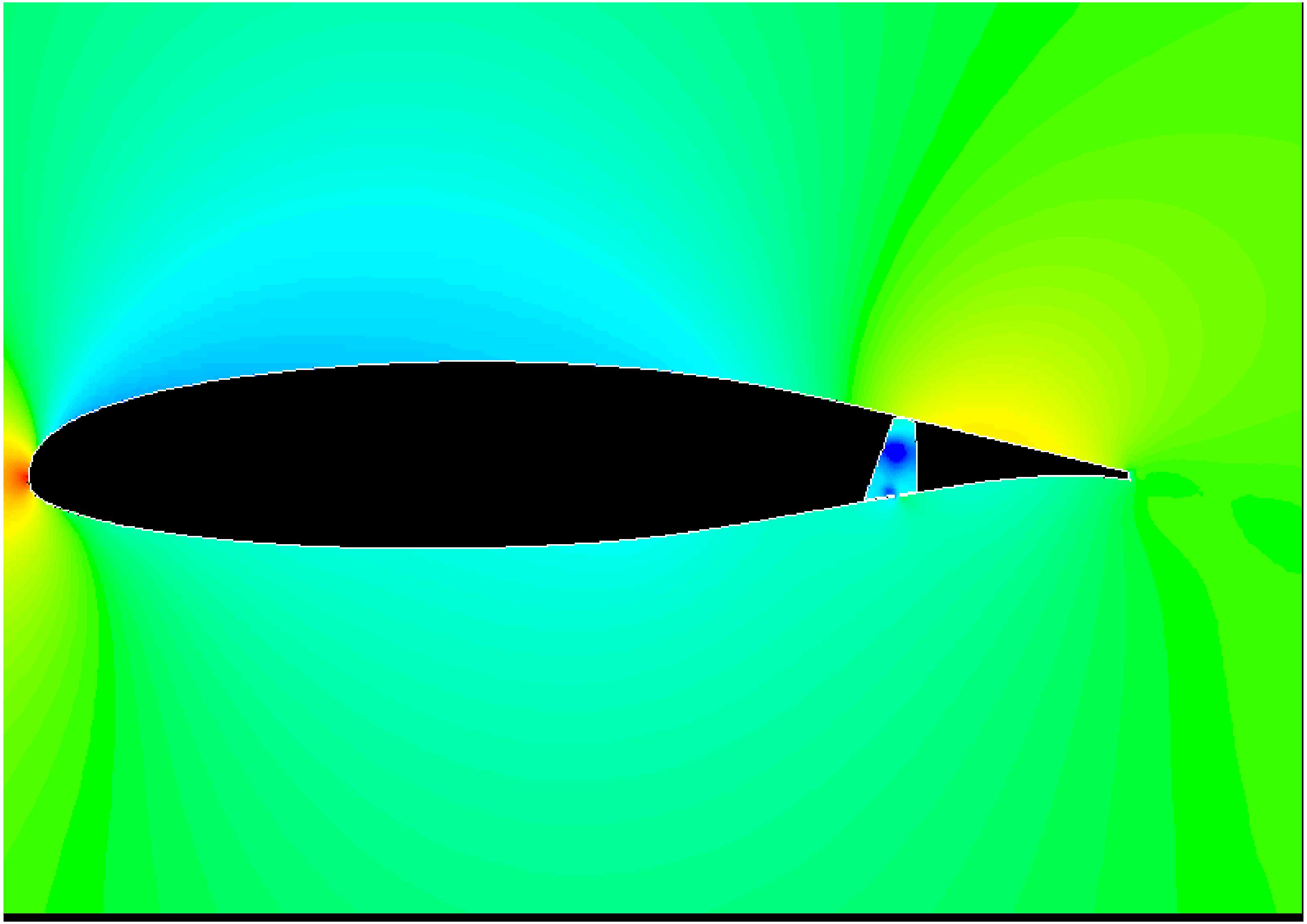
达朗贝尔悖论（佯谬）



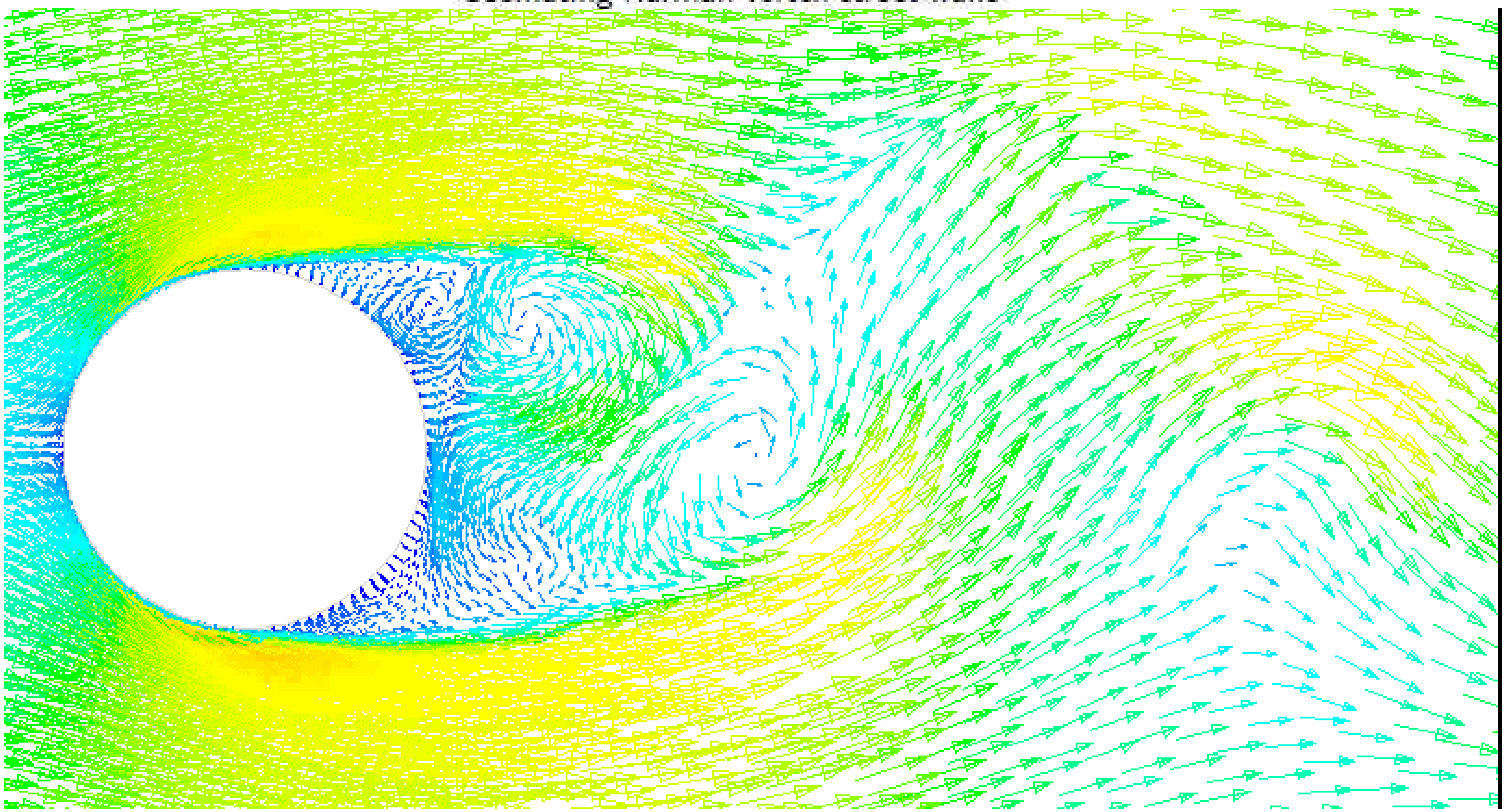
$$p_s = p_0 + \tfrac{1}{2}\rho U^2(1 - 4\sin^2\theta)$$



A comparison of theoretical (inviscid) pressure distribution on the surface of a circular cylinder with typical experimental distribution.



Oscillating Karman vortex street wake



Navier–Stokes equations, named in honor of the French mathematician **L. M. H. Navier** （1758–1836） and the English mechanician **Sir G. G. Stokes** （1819–1903）, who were responsible for their formulation.

纳维（L.Navier，1785—1836，法国数学家）

N-S equations

斯托克斯（G.Stokes，1819—1903，英国力学家））

6.1 Forces Acting on Differential Elements

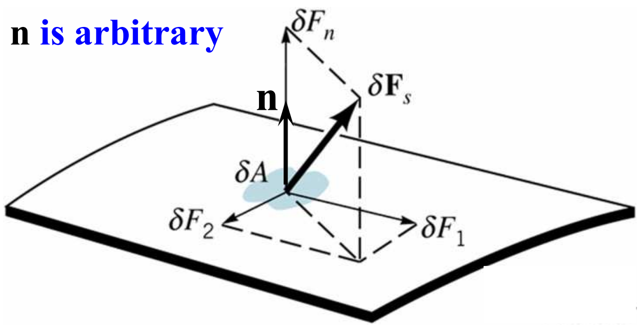
Two types of forces: Body and Surface Forces

Body forces

$$\delta \mathbf{F}_B = \delta m \cdot \mathbf{g}$$

Surface force $\delta \mathbf{F}_S$

n is arbitrary



$$\sigma_n = \lim_{\delta A \to 0} \frac{\delta F_n}{\delta A}$$

Normal stress

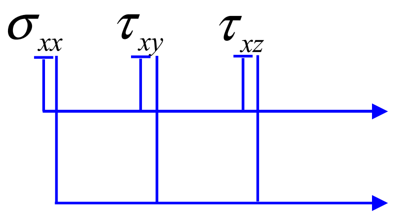
$$\tau_1 = \lim_{\delta A \to 0} \frac{\delta F_1}{\delta A}$$

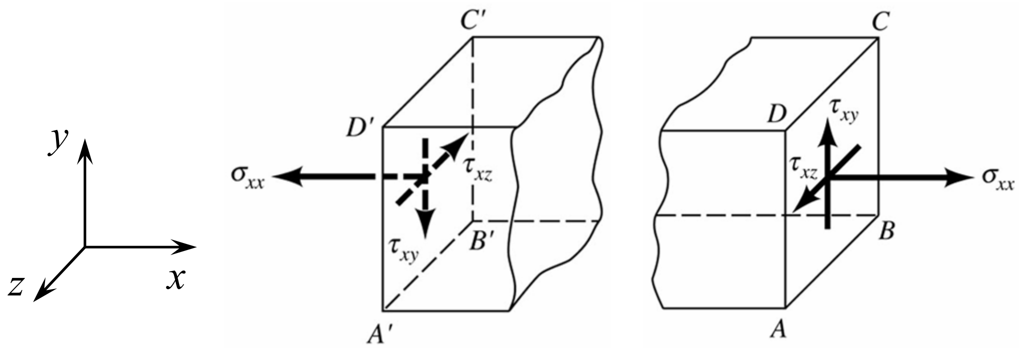
$$\tau_2 = \lim_{\delta A \to 0} \frac{\delta F_2}{\delta A}$$

Shearing stresses

1 subscript can not tell the exact direction

Double subscripts for normal and shearing stresses





6.1 Forces Acting on Differential Elements

Total force on the element in the *x*-direction

$$\delta F_{S,x} = \left(\sigma_{xx} + d\sigma_{xx}\right)dydz - \sigma_{xx}dydz$$
$$+\left(\tau_{yx} + d\tau_{yx}\right)dxdz - \tau_{yx}dxdz + \left(\tau_{zx} + d\tau_{zx}\right)dxdy - \tau_{zx}dxdy$$

$$\delta F_{S,x} = d\sigma_{xx}dydz + d\tau_{yx}dxdz + d\tau_{zx}dxdy$$

$$d\sigma_{xx} = \frac{\partial\sigma_{xx}}{\partial x}dx + \cancelto{0}{\frac{\partial\sigma_{xx}}{\partial y}dy} + \cancelto{0}{\frac{\partial\sigma_{xx}}{\partial z}dz} \qquad d\tau_{yx} = \frac{\partial\tau_{yx}}{\partial y}dy \qquad d\tau_{zx} = \frac{\partial\tau_{zx}}{\partial z}dz$$

$$\delta F_{S,x} = \frac{\partial\sigma_{xx}}{\partial x}dxdydz + \frac{\partial\tau_{yx}}{\partial y}dxdydz + \frac{\partial\tau_{zx}}{\partial z}dxdydz$$

$$\delta F_{S,y} = \frac{\partial\tau_{xy}}{\partial x}dxdydz + \frac{\partial\sigma_{yy}}{\partial y}dxdydz + \frac{\partial\tau_{zy}}{\partial z}dxdydz$$

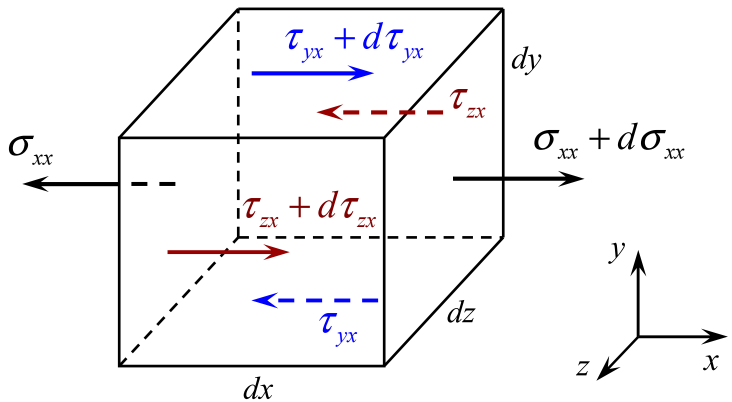
$$\delta F_{S,z} = \frac{\partial\tau_{xz}}{\partial x}dxdydz + \frac{\partial\tau_{yz}}{\partial y}dxdydz + \frac{\partial\sigma_{zz}}{\partial z}dxdydz$$

$$\delta\mathbf{F}_S = \delta F_{S,x}\mathbf{i} + \delta F_{S,y}\mathbf{j} + \delta F_{S,z}\mathbf{k}$$

$$\delta\mathbf{F}_B = \delta m\cdot\mathbf{g} = \delta m\cdot g_x\mathbf{i} + \delta m\cdot g_y\mathbf{j} + \delta m\cdot g_z\mathbf{k}$$

$$\delta\mathbf{F} = \delta\mathbf{F}_S + \delta\mathbf{F}_B = \delta m\cdot\mathbf{a} = \rho dxdydz\cdot\mathbf{a}$$

Stresses in the *x*-direction



$$\frac{\partial\sigma_{xx}}{\partial x}dxdydz + \frac{\partial\tau_{yx}}{\partial y}dxdydz + \frac{\partial\tau_{zx}}{\partial z}dxdydz + \rho dxdydz\cdot g_x$$
$$= \rho dxdydz\cdot a_x$$

x: $$\frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + \frac{\partial\tau_{zx}}{\partial z} + \rho g_x = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

y: $$\frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\sigma_{yy}}{\partial y} + \frac{\partial\tau_{zy}}{\partial z} + \rho g_y = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)$$

z: $$\frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\tau_{yz}}{\partial y} + \frac{\partial\sigma_{zz}}{\partial z} + \rho g_z = \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)$$

应力形式的动量方程

6.2Navier-Stokes (N-S) Equations

$$x:\ \frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}+\rho g_x=\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)$$

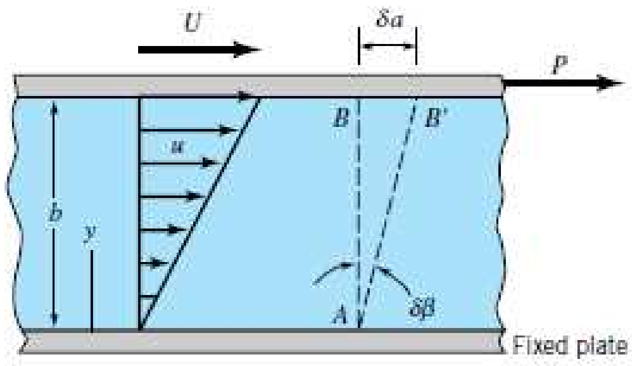
$$y:\ \frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}+\frac{\partial \tau_{zy}}{\partial z}+\rho g_y=\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)$$

$$z:\ \frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \sigma_{zz}}{\partial z}+\rho g_z=\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)$$

Equations of Motion

Stress-Deformation Relationships

Constitutive equations （本构方程） $\tau=\mu\frac{du}{dy}$



$2\varepsilon_{xy}$

$$\tau_{xy}=\tau_{yx}=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\qquad \sigma_{xx}=-p+2\mu\frac{\partial u}{\partial x}$$

$2\varepsilon_{yz}$

$$\tau_{yz}=\tau_{zy}=\mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\qquad \sigma_{yy}=-p+2\mu\frac{\partial v}{\partial y}$$

$2\varepsilon_{zx}$

$$\tau_{zx}=\tau_{xz}=\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\qquad \sigma_{zz}=-p+2\mu\frac{\partial w}{\partial z}$$

Viscous Fluids

$$\frac{\partial}{\partial x}\left(-p+2\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)+\frac{\partial}{\partial z}\left(\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\right)+\rho g_x=\rho(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z})$$

$$\rho(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z})=\rho g_x-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)$$

6.2Navier-Stokes (N-S) Equations

$$\rho(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z})=\rho g_x-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)$$

Incompressible Flows $\nabla\cdot\mathbf{v}=0$ $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$

Navier-Stokes (N-S) Equations

(Viscous and Incompressible Flows)

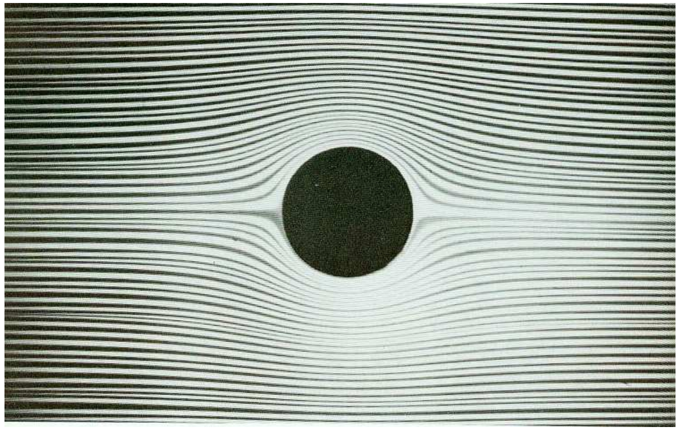
$$x:\ \rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$y:\ \rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\rho g_y+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$z:\ \rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\rho g_z+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

$$\begin{cases} g_x-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2})=\frac{du}{dt} \\ g_y-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2})=\frac{dv}{dt} \\ g_z-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2})=\frac{dw}{dt} \end{cases}$$

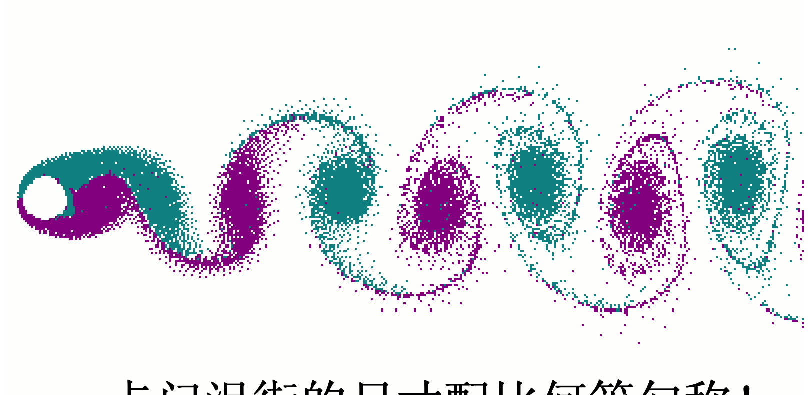
$$\mathbf{g}-\frac{1}{\rho}\nabla p+\upsilon\nabla^2\mathbf{v}=\frac{d\mathbf{v}}{dt}$$



低速绕流圆柱的对称图像的优美

战机翱翔天际的刚毅之美

（优美）对称，比例，简洁



卡门涡街的尺寸配比何等匀称！

大坝喷薄而出的壮阔之美

龙卷风和飞机尾涡在天空绘制的奇美画卷

痛并快乐着

让我们共同追寻流体力学之美!

谢谢大家!